Assignment 12

1. Find an approximation for the minimum of the polynomial $x^4 + x^2 - 40x + 400$ by applying two steps of the golden-mean search starting with the interval [2, 2.5].

```
phi = (sqrt(5) + 1)/2;
f = @(x)(x^4 + x^2 - 40*x + 400);
a = 2;
b = 2.5;
m1 = b - (b - a)/phi;
m2 = a + (b - a)/phi;
[a, m1, m2, b]
          2         2.1910    2.3090    2.5
[f(a), f(m1), f(m2), f(b)]
        340      340.20    341.40    345.31

if f(m1) < f(m2)
  printf( 'Assigning b <- m2\n' );
  b  = m2;
  m2 = m1;
  m1 = b - (b - a)/phi;
else
  printf( 'Assigning a <- m1\n' );
  a  = m1;
  m1 = m2;
  m2 = a + (b - a)/phi;
end
        Assigning b <- m2
[a, m1, m2, b]
          2         2.1180    2.1910    2.3090
[f(a), f(m1), f(m2), f(b)]
        340      339.89    340.20    341.40

if f(m1) < f(m2)
  printf( 'Assigning b <- m2\n' );
  b  = m2;
  m2 = m1;
  m1 = b - (b - a)/phi;
else
  printf( 'Assigning a <- m1\n' );
  a  = m1;
  m1 = m2;
  m2 = a + (b - a)/phi;
end
        Assigning b <- m2
[a, m1, m2, b]
          2         2.0729    2.1180    2.1910

[f(a), f(m1), f(m2), f(b)]
        340      339.84    339.89    340.20
```

2. Find an approximation for the minimum of the polynomial $x^4 + x^2 - 40x + 400$ by applying two steps of the method of successive parabolic interpolation starting with $x_0 = 1.5$, $x_1 = 2.5$ and $x_2 = 2.0$.

```
f = @(x)(x^4 + x^2 - 40*x + 400);
x0 = 1.5;
x1 = 2.5;
x2 = 2.0;
[x0, x1, x2]
         1.5      2.5      2
[f(x0), f(x1), f(x2)]
       347.31   345.31   340

% Find the polynomial interpolating the points
%            (x0, f(x0)), (x1, f(x1)), (x2, f(x2))
%   - You can find the formula in the course notes
% The coefficients go from highest degree (2) to the constant coefficient
p = polyfit( [x0, x1, x2], [f(x0), f(x1), f(x2)], 2 );
% Find the point at which the minimum of that interpolating polynomial is
x3 = -p(2)/(2*p(1));
x3 =  2.0396
[x1, x2, x3]
         2.5      2        2.0396
[f(x1), f(x2), f(x3)]
       345.31   340.00   339.88

p = polyfit( [x1, x2, x3], [f(x1), f(x2), f(x3)], 2 );
x4 = -p(2)/(2*p(1));
[x2, x3, x4]
         2   2.0396   2.0705
[f(x2), f(x3), f(x4)]
       340      339.88   339.85
```

Incidentally, the actual minimum value is 339.84391209576521761.

3. Find an approximation for the minimum of the function –sin(x) + sin(2x) by applying two steps of the golden-mean search starting with the interval [2, 2.5].

```
phi = (sqrt(5) + 1)/2;
f = @(x)(-sin(x) + sin(2*x));
a = 2;
b = 2.5;
m1 = b - (b - a)/phi;
m2 = a + (b - a)/phi;
[a, m1, m2, b]
       2.0000   2.1910   2.3090   2.5000
[f(a), f(m1), f(m2), f(b)]
      -1.6661  -1.7597  -1.7352  -1.5574

if f(m1) < f(m2)
  printf( 'Assigning a <- m1\n' );
  b = m2;
  m2 = m1;
  m1 = b - (b - a)/phi;
else
  printf( 'Assigning a <- m1\n' );
  a = m1;
  m1 = m2;
  m2 = a + (b - a)/phi;
end
    Assigning a <- m1
[a, m1, m2, b]
       2.0000   2.1180   2.1910   2.3090
[f(a), f(m1), f(m2), f(b)]
      -1.6661  -1.7427  -1.7597  -1.7352

if f(m1) < f(m2)
  printf( 'Assigning a <- m1\n' );
  b = m2;
  m2 = m1;
  m1 = b - (b - a)/phi;
else
  printf( 'Assigning a <- m1\n' );
  a = m1;
  m1 = m2;
  m2 = a + (b - a)/phi;
end
    Assigning a <- m1
[a, m1, m2, b]
       2.1180   2.1910   2.2361   2.3090
[f(a), f(m1), f(m2), f(b)]
      -1.7427  -1.7597  -1.7580  -1.7352
```

4. Find an approximation for the minimum of the function –sin(x) + sin(2x) by applying two steps of the method of successive parabolic interpolation starting with $x_0 = 1.5$, $x_1 = 2.5$ and $x_2 = 2.0$.

```
f = @(x)(-sin(x) + sin(2*x));
x0 = 1.5;
x1 = 2.5;
x2 = 2.0;
[x0, x1, x2]
       1.5000   2.5000   2.0000
[f(x0), f(x1), f(x2)]
      -0.85637  -1.55740  -1.66610

% Find the interpolating polynomial
p = polyfit( [x0, x1, x2], [f(x0), f(x1), f(x2)], 2 );
% Find the point where the minimum of that interpolating polynomial is
x3 = -p(2)/(2*p(1));
[x1, x2, x3]
       2.5000   2.0000   2.1908
[f(x1), f(x2), f(x3)]
      -1.5574  -1.6661  -1.7597

p = polyfit( [x1, x2, x3], [f(x1), f(x2), f(x3)], 2 );
x4 = -p(2)/(2*p(1));
[x2, x3, x4]
       2.0000   2.1908   2.2025

[f(x2), f(x3), f(x4)]
      -1.6661  -1.7597  -1.7601
```

5. Find an approximation of the minimum of the polynomial $x^2 + 2y^2 - xy - 8x + 5y - 1$ by applying the Hooke-Jeeves method starting with $x = y = 0$ and $h = 1$ and continuing until $h < 0.25$.

We underline the minimum value at each step. Let $\mathbf{s}_1$ and $\mathbf{s}_2$ be the two canonical basis vectors.

```
s1 = [1 0]';
s2 = [0 1]';
x0 = [0 0]';
f = @(x)( x(1)^2 + 2*x(2)^2 - x(1)*x(2) - 8*x(1) + 5*x(2) - 1 );
[f(x0 - s1), f( x ), f(x0 + s1)]
        8     -1      -8
[f(x0 + s1-s2), f(x0 + s1), f(x0 + s1+s2)]
      -10     -8      -2

% Move in the direction of s1-s2:
% We note f(x0 + n*(s1-s2)) is minimized when n == 2:
[f(x0 + (s1-s2)), f(x0 + 2*(s1-s2)), f(x0 + 3*(s1-s2))]
      -10    -11      -4

x1 = x0 + 2*(s1 - s2)
    x1 =  2
         -2
[f(x1 - s1), f(x1), f(x1 + s1)]
      -8     -11     -12
[f(x1 + (s1-s2)), f(x1 + s1), f(x1 + (s1+s2))]
      -4     -12     -16

% Move in the direction of s1+s2:
% We note f(x1 + n*(s1+s2)) is minimized, again, when n == 2:
[f(x1 + (s1+s2)), f(x1 + 2*(s1+s2)), f(x1 + 3*(s1 + s2))]
     -16     -17     -14

x2 = x1 + 2*(s1+s2)
    x2 =  4
          0
[f(x2 - s1), f(x2), f(x2 + s1)]
     -16     -17     -16
[f(x2 - s2), f(x2), f(x + s2)]
     -16     -17     -14
% Thus, we divide h by two:
s1 = s1/2;
s2 = s2/2;
[f(x2 - s1), f(x2), f(x2 + s1)]
     -16.75  -17     -16.75
[f(x2 - s2), f(x2), f(x2 + s2)]
     -17     -17     -16
% Still a minimum, so divide h by two, again:
s1 = s1/2;
s2 = s2/2;
[f(x2 - s1), f(x2), f(x2 + s1)]
     -16.938 -17     -16.938
[f(x2 - s2), f(x2), f(x2 + s2)]
     -17.125 -17     -16.625

% Move in the direction of -s2
[f(x2 + (-s2)), f(x2 + 2*(-s2)), f(x2 + 3*(-s2))]
     -17.125 -17     -16.625

x3 = x2 + 1*(-s2)
      x2 =  4
         -0.25
[f(x - s1), f(x), f(x + s1)]
     -17.125 -17.125 -17
[f(x - s2), f(x), f(x + s2)]
     -17     -17.125 -17
% Thus, divide h by two...and for now, we are done
```

5. Find an approximation of the minimum of the polynomial $x^2 + 2y^2 - xy - 8x + 5y - 1$ by applying one step of Newton's method in n dimensions for finding extrema starting with $x = 4$ and $y = 0$.

The gradient is $(2x - y - 8, 4y - x + 5)^T$, and so the Jacobian is the matrix is calculated from this:

```
     f = @(x)( x(1)^2 + 2*x(2)^2 - x(1)*x(2) - 8*x(1) + 5*x(2) - 1 );
 gradf = @(x)( [2*x(1) - x(2) - 8; 4*x(2) - x(1) + 5] );
Jgradf = @(x)( [2 -1; -1 4] );
x0 = [4 0]';
f(x0)
        -17

dx0 = Jgradf( x0 ) \ -gradf( x0 )
    dx0 = -0.1428571428571428
          -0.2857142857142858

x1 = x0 + dx0
    x1 =  3.857142857142857
         -0.2857142857142858

f(x1)
        -17.14285714285714
```

6. Find the gradient of the polynomial $x^2 + 2y^2 - xy - 8x + 5y - 1$ at the point $x = 4$ and $y = 0$ and convert this into a polynomial in a single variable. The minimum of a quadratic polynomial $ax^2 + bx + c$ is $-b/2a$, so use this to determine the next point. Calculate the gradient at this second point.

We have:

```
    f = @(x)( x(1)^2 + 2*x(2)^2 - x(1)*x(2) - 8*x(1) + 5*x(2) - 1 );
gradf = @(x)( [2*x(1) - x(2) - 8; 4*x(2) - x(1) + 5] );
x0 = [4 0]';
f(x0)
      -17
gradf( x0 )
       0
       1
```

Thus, we have (4, 0) – s(0, 1) so we have $f((4, -s)^T) = 4^2 + 2(-s)^2 - 4(-s) - 8\cdot 4 + 5(-s) - 1 = 2s^2 - s - 17$. This has a minimum at 0.25, so our next approximation is (4, 0) – 0.25(0, 1) = (4, –0.25)

```
x1 = [4, -0.25]';
f(x1)
    -17.125
gradf(x1)
     0.25
     0
```

7. Are the two gradient vectors in Question 6 reasonably orthogonal?

We note that the two gradients are orthogonal.